Theory Camp: Microeconomics Essentials

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Theory of Consumer Behaviour

How consumers allocate income among different goods/services to maximise their wellbeing. 3 pillars:

1. Consumer preferences
2. Budget constraints
3. Choice!

Assumptions (1)

1. **Completeness**: $\forall x \epsilon \mathbb{X}$, either *xRy* or *yRx* or both. We can *rank* bundles.
2. **Transitivity**: $\forall x \epsilon \mathbb{X}$, if *xRy* is true and *yRz* is true, then *xRz* is true. Be consistent with rankings.
3. **More is better than less**, no intersecting indifference curves or corner solutions (monotonicity, non-satiation)
4. **Convexity**- diminishing marginal returns, diminishing rate of substitution. (MRS* $\downarrow$ as you move along an indifference curve)

Assumptions (2)

At the optimal consumption bundle $(x*, y*)$ the budget line and indifference curve are tangent to one another. That is, the MRS between two goods is exactly equal to the price ratio $(MRS = p_x/p_y)$. At this point, $MB = MC$.

*MRS = the marginal rate of substitution, how much of good y we are willing to give up in exchange for consuming more of good x

Derive the budget line (1)

Equation of a straight line: $y = mx + c$ (See Year 9-10 maths textbook if you are lost- Heinemann is a good one)
The budget relation tells us we can't spend more money on the 2 goods, x and y, than we have. That is, $I = p_x x + p_y y$.

To plot this relation onto a graph showing the feasible combinations of *x* and *y*, we turn this equation into a straight line.
$p_y y = I - p_x x$
$y = \frac{I - p_x x}{p_y}$
The intercept is: I/p_y
The slope is: $-p_x/p_y$ (price ratio!)*
The x-intercept is: I/p_x

*The slope, or gradient, of any function describes the rate of change of one variable (here it is *y)* with respect to another (*x*). It is also calculated by using $\frac{rise}{run}$ and shows how the variable you are measuring on the *x*-axis of your graph affects the one you are measuring on the *y*-axis.

Derive the budget line (2)

You can now easily draw the budget line. The budget line acts as a *constraint* and shows all of the feasible combinations of the 2 goods, x and y, given the amount of money the consumers has been endowed with (to spend).

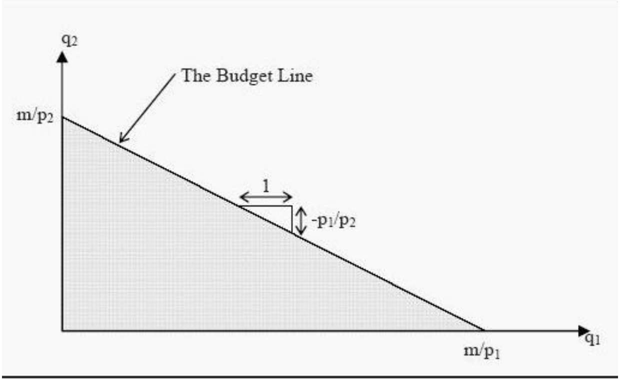


Image Source: tutorhelpdesk.com (2014)

Derive Indifference Curves (1)

An indifference curve, $U(x, y)$, shows all the possible consumption bundles of the 2 goods, x and y, that give the consumer the same level of utility.

That means, along an indifference curve, there is no change in utility. We can use **total differentiation** to calculate the gradient along an indifference curve.

Differentiation Rules

Total differentiation (for ≤ 2 variables):

To derive $f(t, x, y)$ with respect to t, must account for the fact that the function is affected by two other variables, x and y, who may also change when t changes. In turn, each of those changes may further affect t.

$\frac{df}{dt} = \frac{\partial f}{\partial t}\frac{dt}{dt} + \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial x}\frac{dy}{dt}$

Partial Differentiation:

When we hold everything else constant (HAEC) and ignore how other variables affect f, we only derive the function with respect to a single variable (denoted by ∂).

E.g. $U(x, y) = xy^2 + y$. Then $\frac{\partial U}{\partial y} = 2xy + 1$ and $\frac{\partial U}{\partial x} = y^2 + y$.

Derive Indifference Curves (2)

From the total differentiation rule, we can infer the following relationship about any changes in the function $f(t, x, y)$:
$\Delta f = \frac{\partial f}{\partial t}\Delta t + \frac{\partial f}{x}\Delta x + \frac{\partial f}{\partial y}\Delta y$

That is, any changes in the dependent variable (or function), $f(t, x, y)$, result from changes in any of the independent variables (or variables that the function depends on- *t, x* and *y)* and how much each one affects f HAEC.

Along an indifference curve, there is no change in utility, so we can write use the above in the context of the indifference curve:
$\Delta U(x, y) = \frac{\partial U(x,y)}{\partial x}\Delta x + \frac{\partial U(x,y)}{\partial y}\Delta y$

Derive Indifference Curves (3)

$\Delta U(x, y) = \frac{\partial U(x,y)}{\partial x}\Delta x + \frac{\partial U(x,y)}{\partial y}\Delta y$

BUT along an indifference curve, $\Delta U(x, y) = 0$!

We also know that $\frac{\partial U(x,y)}{\partial x} = MU_x$ and $\frac{\partial U(x,y)}{\partial y} = MU_y$.

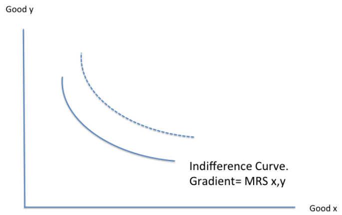
Derive Indifference Curves (4)

So: $0 = MU_x \Delta x + MU_y \Delta y$

$MU_y \Delta y = -MU_x \Delta x$

$\frac{\Delta y}{\Delta x} = -\frac{MU_x}{MU_y} = MRS_{x,y}$

This is the gradient of the indifference curve, showing the rate at which a consumer is willing to exchange some of good x for a unit of good y.



Optimal bundle

So at our optimal consumption point, $(x*, y*)$, having taken into account our preferences and our budget, the following holds:

$$\frac{\Delta y}{\Delta x} = -\frac{MU_x}{MU_y} = MRS_{x,y} = -\frac{p_x}{p_y}$$

The gradient of the indifference curve is the same as that of the budget line. A consumer is doing *the best they can* given their constraints.

Extension: Producer Theory (1)

The firm can use two inputs, capital and labour (K and L), to produce output. Similar to the budget line, an **isocost line** shows all the bundles of K and L that can be purchased at a given total cost. Along an **isoquant**, similar to an indifference curve, all possible combinations of the two inputs, K and L, are shown that prouce the same level of output.

You can plot capital along the y-axis and labour along the x-axis. Using the same methodology employed in consumer theory, you will find that at the optimal mix of inputs $(l*, k*)$ the slopes of the isocost line and the isoquant line are equal.

Extension: Producer Theory (2)

Isocost line: $C = wL + rK$.
Slope: $-\frac{w}{r}$
where $w = p_L$ (wages) and $r = p_k$ (interest rate).

Along an isoquant:

$$\Delta Q = 0 = \frac{\partial Q}{\partial L}\Delta L + \frac{\partial Q}{\partial K}\Delta K = MP_L \Delta L + MP_K \Delta K.$$

Slope: $\frac{\Delta K}{\Delta L} = -\frac{MP_L}{MP_K} = MRT_{l,k}$

At optimal production mix: $-\frac{w}{r} = MRT_{l,k}$. In other words, at the optimal input mix, the price ratio is the same as that of the tradeoff a producer faces when comparing two inputs.

Substitution and Income Effect (1)

Total Effect = SE + IE.

A change in the price of a good has two effects; *e.g.* price $\downarrow$ ($p_x \downarrow$)

1. Substitution Effect:
Consumers tend to buy more of the good that is now cheaper (WTP$\uparrow$) and less of the good that is now relatively more expensive (WTP $\downarrow$). This response to a **change in the relative prices** of goods appears as a *movement along the same indifference curve* as we switch from a bundle which contains more *y* and less *x* to a bundle that contains less *y* and more *x.*

- $QD_y \downarrow QD_x \uparrow$ U(x,y) is held constant
- Slope changes!

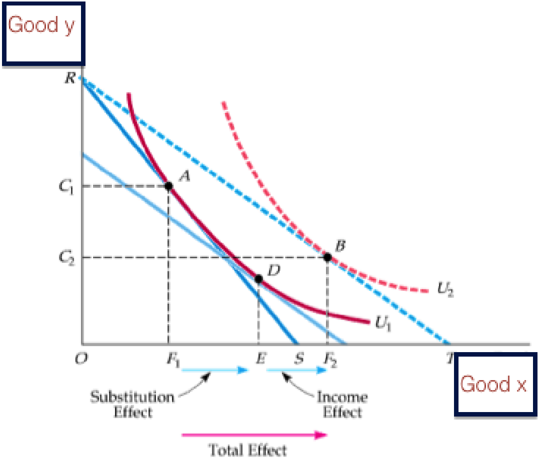
Substitution and Income Effect (2)

2. Income Effect:
One of the goods is now cheaper so consumers' real purchasing power has increased and they feel wealthier. Because they can now afford more, consumers will purchase more of both goods (WTP↑)(WTP↑) . The change in demand for both goods due to a **change in real income** is represented as a *shift of the budget constraint outwards and the abililty to move to a higher indifference curve*.

- $QD_x \uparrow QD_y \uparrow$ $\frac{p_x}{p_y}$ is held constant
- Intercept changes!

Substitution and Income Effect (2)

e.g. price ↓ (p_x ↓)
Example and picture adapted from Pindyck & Rubinfeld (2008)



Substitution and Income Effect (3)

- The consumer is initially at A, on budget line RS. As $p_x \downarrow$, consumption increases by F1F2 as the consumer moves to B.
- The substitution effect F1E (associated with a move from A to D) changes the relative prices of x and y but keeps real income (utility) constant.
- The income effect EF2 (associated with a move from D to B) keeps relative prices constant but increases purchasing power. x is a normal good because the income effect EF2 is positive.

Substitution and Income Effect (4)

- In production, these are called the *scale effect* (change in overall affordability- demand more/less of both inputs) and the *substitution effect* (change in relative price of inputs)
- In our previous example, demand for x increased due to the income effect and decreased due to the substitution effect. Which one dominates? This will affect where the new optimal bundle lies. With a good, we can infer whether SE>IE or IE<SE based on whether the good is inferior or normal. When considering labour supply in the producer's input mix, it is uncertain- more information must be given to you to figure it out.

LaGrangean Method of Constrained Optimisation (1)

The LM converts a *constrained maximisation* problem into an *unconstrained maximisation* problem by using a LaGrangean multiplier. The LM is one of *many ways* to solve a constrained optimisation problem.

- Assume the problem is an equality constraint problem
- e.g. $I = p_x x + p_y y$
- (If the sign were $\geq$, we would use another method of solving called the Kuhn-Tucker method..)

LaGrangean Method of Constrained Optimisation (2)

Theorem

Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ and $g : \mathbb{R}^n \rightarrow \mathbb{R}$, where both are C^1 on $\mathbb{R}^n$. Then there exists a vector, $\lambda \epsilon \mathbb{R}^k$, such that $Df(x) + \lambda Dg'(x) = 0$

- f: objective function (what you are looking to maximise/minimise)
- g: constraint
- C^1: continuous over the $\mathbb{R}^n$ domain

If the constraint is continuous (the limit exists at all points), partial/derivatives exist and are also continous.

LaGrangean Method of Constrained Optimisation (3)

What do you mean 'a limit might not exist'? Check this website:

http://www.wyzant.com/resources/lessons/math/calculus/differentiation

LaGrangean Method of Constrained Optimisation (4)

1. Set constraint=0. Form the LaGrangean function:

$$\max_{x,y} U(x,y) \text{ s.t. } I = p_x x + p_y y$$

$$\mathcal{L} = U(x,y) + \lambda(I - p_x x - p_y y)$$

Important: The sign of λ is always the same as the sign of the limiting constraint, I.

2. Find stationary points of $\mathcal{L}$ by partially differentiating with respect to each argument- obtain 3 FOC's (first order conditions), $\mathcal{L}_x$, $\mathcal{L}_y$ and $\mathcal{L}_\lambda$.

2a. Set each FOC =0.

3. Solve FOC's 1 & 2 ($\mathcal{L}_x$, $\mathcal{L}_y$) and eliminate λ to find a relation between x and y. (Tradeoff at optimal point)

3a. Sub this into FOC 3 ($\mathcal{L}_\lambda$) to find demand for each good at the optimal consumption bundle.

LaGrangean Method of Constrained Optimisation (5)

What about λ? Should I go back and find it?

Maybe! You are able to (by following step 3a with λ as well as with *x* and *y)*– Depends what kind of information you want. In general, the Lagrange multiplier, λ, measures the approximate change in the value of the optimised function in response to a small change in the constraint.

For example if we are utility maximising subject to a budget constraint, it measures the approximate change in the number of goods consumed at the optimal level (change in our bundle $(x*, y*)$) when we are given an extra dollar of income. In this context, it represents the marginal utility of money..!

Example: Tute Question W2Q5 (1)

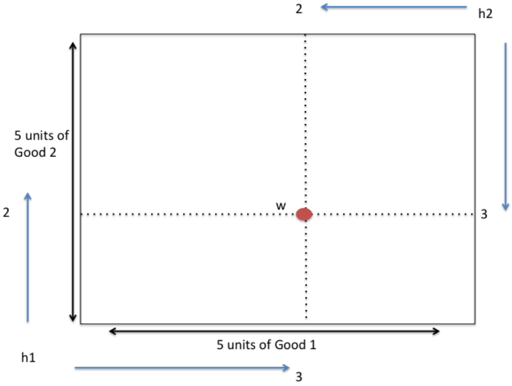
Let two consumers have preferences described by the utility function $U^h = log(x_1^h) + log(x_2^h)$ $h = 1, 2$.

	Good 1	Good 2
Consumer 1	3	2
Consumer 2	2	3

i. Calculate the consumers' demand functions.
ii. Using good 2 as the numeraire, find the equilibrium price of good 1.
iii. Hence, find the equilibrium levels of consumption. Show that the consumers' indifference curves are tangential at equilibrium.

Example: Tute Question W2Q5 (2)

Rember the Edqeworth Box rule:
Any number of consumers can never spend more than they are endowed with in total!



Example: Tute Question W2Q5 (3)

i. Calculate the consumers' demand functions.

Form the $\mathcal{L}$:

$$\max_{x_1^h, x_2^h} log(x_1^h) + log(x_2^h) \text{ s.t. } p_1 x_1^h + p_2 x_2^h = p_1 w_1 + p_2 w_2$$

$$\mathcal{L} = log(x_1^h) + log(x_2^h) + \lambda(p_1 w_1 + p_2 w_2 - p_1 x_1^h - p_2 x_2^h)$$

Example: Tute Question W2Q5 (4)

Find FOC:

(For a revision of differentiation formulas, see http://www.s-cool.co.uk/a-level/maths/integration/revise-it/introduction. For a revision of differentiation rules, see http://www.cs.gmu.edu/cne/modules/dau/calculus/derivatives/deriv_laws_bdy.html)

$$\mathcal{L}_{x_1^h} = \frac{1}{x_1^h} - \lambda p_1$$

$$\mathcal{L}_{x_2^h} = \frac{1}{x_2^h} - \lambda p_2$$

$$\mathcal{L}_{\lambda} = p_1 w_1 + p_2 w_2 - p_1 x_1^h - p_2 x_2^h$$

Example: Tute Question W2Q5 (4)

Set each FOC=0 to obtain:

$\frac{1}{x_1^h} = \lambda p_1$

$\frac{1}{x_2^h} = \lambda p_2$

$p_1 w_1 + p_2 w_2 = p_1 x_1^h + p_2 x_2^h$

Checkpoint: Is FOC3 the same as your original constraint?

Example: Tute Question W2Q5 (5)

- Make λ the subject in FOC1 and FOC2. Eliminate it to obtain: $\frac{x_1^h}{x_2^h} = \frac{p_2}{p_1}$.
  - This relation explains exactly how the two goods are being 'traded-off' by consumers *at the optimal point of consumption.*
  - It looks like the price ratio, which makes sense. (Might not always be exactly that though!)
- Now you can easily substitute $x_2^h = \frac{x_1^h p_1}{p_2}$ into FOC3 (budget constraint) and make x_1^h the subject to find the demand for x_1^h. Do the same for x_2^h.

Example: Tute Question W2Q5 (6)

- Any consumer's demand for each good is: $x_1^h = \frac{p_1 w_1^h + p_2 w_2^h}{2p_1}$ and $x_2^h = \frac{p_1 w_1^h + p_2 w_2^h}{2p_2}$
- To find individual demand functions, sub in the known endowment values ($w_1^1 = 3$, $w_2^1 = 2$ and $w_1^2 = 2$, $w_2^2 = 3$) for each consumer to get:
- $x_1^1 = \frac{3p_1 + 2p_2}{2p_1}$ and $x_2^1 = \frac{3p_1 + 2p_2}{2p_2}$
- $x_1^2 = \frac{2p_1 + 3p_2}{2p_1}$ and $x_2^2 = \frac{2p_1 + 3p_2}{2p_2}$

Example: Tute Question W2Q5 (7)

ii. Using good 2 as the numeraire, find the equilibrium price of good 1.

Remember, we can't consumer more than we have so at equilibrium, $x_1^1 + x_1^2 = w_1^1 + w_1^2$ and $x_2^1 + x_2^2 = w_2^1 + w_2^2$.

For good 1, sub in each consumer's known demands:
$\frac{3p_1+2p_2}{2p_1} + \frac{2p_1+3p_2}{2p_1} = 3 + 2 = 5$

Set the price of good 2 as the numeraire. *Force* $p_2 = 1$.

Then the equilibrium condition becomes: $\frac{3p_1+2}{2p_1} + \frac{2p_1+3}{2p_1} = 5$ and we can easily solve for p_1.

Example: Tute Question W2Q5 (8)

$\frac{3p_1+2+2p_1+3}{2p_1} = 5$

$\frac{5p_1+5}{2p_1} = 5$

$\frac{1p_1+1}{2p_1} = 1$

$\frac{1}{2} + \frac{1}{2p_1} = 1$

$\frac{1}{2p_1} = \frac{1}{2}$ So $p_1 = 1$.

Example: Tute Question W2Q5 (8)

iii. Hence, find the equilibrium levels of consumption.

If $p_1 = p_2 = 1$, then:

- $x_1^1 = \frac{3p_1+2p_2}{2p_1} = \frac{5}{2}$ and $x_2^1 = \frac{3p_1+2p_2}{2p_2} = \frac{5}{2}$
- $x_1^2 = \frac{2p_1+3p_2}{2p_1} = \frac{5}{2}$ and $x_2^2 = \frac{2p_1+3p_2}{2p_2} = \frac{5}{2}$

Example: Tute Question W2Q5 (9)

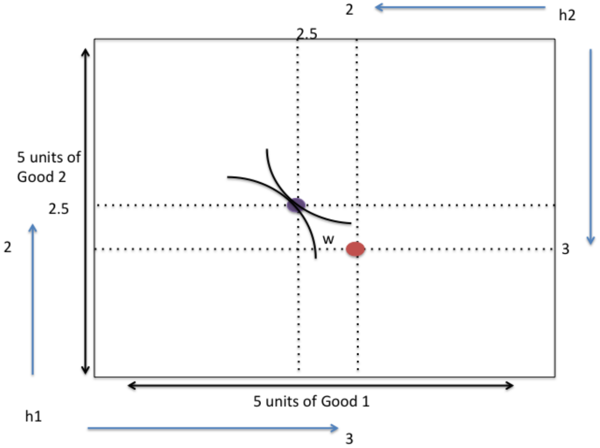
Show that the consumers' indifference curves are tangential at equilibrium.

The gradient of the indifference curve is $MRS^h_{1,2} = \frac{\partial U^h \backslash \partial x^h_1}{\partial U^h \backslash \partial x^h_2}$ for any consumer h.

For this utility function, $\frac{\partial U}{\partial x^h_i} = \frac{1}{x^h_i}$

So $MRS^h_{1,2} = \frac{1/x^h_1}{1/x^h_2} = \frac{x^h_2}{x^h_1} = \frac{5/2}{5/2} = 1$ for each consumer. Both consumers have the same MRS so their indifference curves are tangential.

Example: Tute Question W2Q5 (10)



Through a process of voluntary exchange, consumers have traded away from intitial endowment to maximise their utility.